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SUBJECT

BACKGROUND GENERATED BY MOVING BEAMS ON MIRRORS WITH
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Background Generated by Moving Beams on Mirrors with Temperature Gradients and Dust Contamination.

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I. Introduction:

The aim of this report is to illustrate under what conditions a telescope system incorporating active optics or beam steering mirrors can induce an unacceptable IR background noise into a SOFIA science instrument.

This report is based on two previous reports, one by Wright (198?), and another by Davidson and Erickson (1988); these are attached in Appendices A, and B, respectively. As explained in these reports, infrared telescopes use "choppers" to cancel background sky and telescope emission from the emission of an astronomical source. The result is an ac-signal at the frequency of the chop with thermal "shot" noise contributions from the astronomical source, sky and telescope. At IR wavelengths the Noise Equivalent Power (NEP) due to the thermal shot noise from the telescope and sky is approximately one-millionth of their thermal emission. Many interesting astronomical sources have signals on the order of this NEP. Thus it is important to (1) subtract out the background emission and (2) not introduce any further noise while doing so. The report by Davidson and Erickson illustrated that a "nodding plus chopping" routine can be successfully employed using a chopping secondary in a Cassegrain telescope configuration to cancel the sky and telescope background emission down to the NEP level of the sky and telescope. This idealized subtraction was achieved without introducing noise, since the chop and nod mechanisms were assumed perfect and so each placement of the beam in the chopping/nodding sequence was reproduced exactly. However, in practice such accuracy is not possible; low level random motion of the beam on the primary (or any other mirror surface) occurs, introducing noise if the mirror surface has temperature gradients, defects or dust contamination. Active optics will also introduce noise to the signal in a similar way. We will qualitatively and quantitatively discuss the noise introduced by beam motion on a telescope mirror surface in the following sections. Sections II and III will estimate the noise generated by mirrors with temperature gradients and dust contamination, respectively, and section IV will give examples illustrating how to use the information derived in sections II and III.

II. Background Induced by Temperature Gradient on a Mirror

As given in the report by Davidson and Erickson, the power falling on a detector from a mirror in the optical path of the telescope is given by:-

$$P_m = \epsilon_m \cdot B(\nu, T_m) \cdot \overline{\tau} \cdot \Delta\nu \cdot t \quad (1)$$

where ϵ_m is the emissivity of the mirror, $B(\nu, T_m)$ is the Planck function which is a function of the frequency, ν , of the radiation being observed and mirror temperature, T_m , $\overline{\tau}$ is the optical throughput of the telescope, $\Delta\nu$ is the frequency bandpass of the detector, and t is the transmittance of the science instrument. The throughput of a telescope system is a constant along the light path. At the primary mirror $\overline{\tau} = A_p \cdot \Omega_p$ where A_p is the area of the primary and Ω_p is the solid angle subtended by a detector's beam on the sky. At the mirror in question $\overline{\tau} = A_m \cdot \Omega_m$ where A_m is the area of this mirror and Ω_m is the apparent solid angle subtended by the detector at this mirror.

Let us consider the case where the mirror has a uniform emissivity (ie., has no defects or dust contamination), but it does have a temperature gradient. Then if the beam moves on the mirror the change in background power at the detector would be:-

$$\Delta P_m = \epsilon_m \cdot \{B(\nu, T_m + \Delta T) - B(\nu, T_m)\} \cdot \Delta A \cdot \Omega_m \cdot \Delta \nu \cdot t \quad (2)$$

where ΔT is the change in the mean temperature of the mirror within the beam and ΔA is the crescent area change in mirror illumination. For $\Delta T/T_m \ll 1$ the following approximation holds:-

$$\Delta P_m = P_m \cdot [x/(1 - \exp(-x))] \cdot [\Delta T/T_m] \cdot [\Delta A/A_m] \quad (3a)$$

$$= P'_m \cdot [x \cdot e^x / (e^x - 1)^2] \cdot [\Delta T/T_m] \cdot [\Delta A/A_m] \quad (3b)$$

where we have shown explicitly the functional dependence on temperature using $x = [14400/\lambda T_m]$, where λ is wavelength in microns, and $P_m = P'_m \cdot [1/(e^x - 1)]$.

Such a change in the background will produce offsets which simulate noise if the beam moves on the mirror in a random manner at the frequency of the chopper. Hence, we must require for this random motion that:-

$$|\Delta P_m| < (1/2) \text{NEP} / \sqrt{s} \quad (4)$$

where NEP is the noise from the sky and telescope (ie., $\text{NEP}^2 = \text{NEP}_{\text{tel}}^2 + \text{NEP}_{\text{sky}}^2$), and s is a time interval on the order of a chop period. The factor of a half insures that the noise generated by the random beam motion can be ignored. (Recall that noise sources are summed in quadrature.) At 30 microns, for broadband observations, the NEP typically equals about $10^{-15} \text{ W}/\sqrt{\text{Hz}}$, and a chopper frequency of about 20Hz is commonly used. By combining equations 3b and 4 we get:-

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < [\text{NEP} / (2 P'_m \sqrt{s})] \cdot [(e^x - 1)^2 / (x e^x)] \quad (5a)$$

$$= 1.6 \times 10^{-6} [(e^x - 1)^2 / (x e^x)] \quad (5b)$$

Relation 5b was derived for diffraction limited observations using a $\Delta\lambda/\lambda = 0.2$ bandpass centered at 30 microns, a detector system transmittance of 0.3, and a mirror emissivity of 0.05. No assumption was made about the temperature of the mirror. If we assume the temperature of the mirror is 240K, then 5b becomes:-

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 4.4 \times 10^{-6} \quad (6a)$$

If we assume the temperature of the mirror is 77K (nitrogen cooled) or 4K (helium cooled mirror), then

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 1.3 \times 10^{-4} \quad T_m = 77\text{K} \quad (6b)$$

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 1.7 \times 10^{44} \text{ !!!!} \quad T_m = 4\text{K} \quad (6c)$$

Relations 6b and 6c illustrate that $[(e^x - 1)^2 / (x e^x)]$ is a strong function of T_m and that helium cooled mirrors do not present a background noise problem.

Relation 6a should be used to judge all warm mirror configurations using active or moving optics. [As an aside, remember that the ΔA term in the above equations is only that area change due to random beam motion at the frequency of the chop. The constant beam motion due to the chop itself, for example, only contributes a constant dc-offset which can be subtracted out. And motions of the beam at frequencies other than the chop frequency are not integrated into the astronomical signal.]

III. Background Induced by Dust Contamination on a Mirror

Now consider a mirror with no temperature gradients but which has random emissivity irregularities (ie., dust and defects). The report by Wright showed that the power emitted by a dust grain on the mirror into a detector can be given by:-

$$P_{\text{dust}} = \pi a^2 Q \cdot \Omega_m \cdot B(\nu, T_m) \cdot \Delta \nu \cdot t \quad (7)$$

where the radius and emissivity of the dust grain are a and Q , respectively. (Here we have assumed that the dust on the mirror has the same temperature as the mirror.) If the dust on the mirror has a random distribution, such that the number of grains with radii between a and $a+da$ within an area ΔA is given by $\Delta A \cdot n(a) da$, then the background power at the detector due to these grains would be:-

$$P_{\Delta A} = \Omega_m \cdot B(\nu, T_m) \cdot \Delta \nu \cdot t \cdot \int \Delta A \cdot \pi a^2 Q \cdot n(a) da \quad (8a)$$

$$= P_m \cdot [\Delta A \cdot \pi / \epsilon_m A] \cdot \int a^2 Q \cdot n(a) da \quad (8b)$$

If the beam moves on the surface of the mirror, different dust samples will become visible to the detector, thus causing background offsets, ΔP_m . The magnitude of ΔP_m will equal the standard deviation of $P_{\Delta A}$, which can be estimated since the dust grains are distributed randomly on the mirror, so the standard deviation of the number of grains with radii between a and $a+da$ within an area ΔA is given simply by $(\Delta A \cdot n(a) da)^{1/2}$. Thus by summing in quadrature the standard deviation contributions of all the grains within ΔA :-

$$\Delta P_m = \sigma(P_{\Delta A}) = P_m \cdot [\pi / \epsilon_m] \cdot [\Delta A / A^2]^{1/2} \cdot [\int a^4 Q^2 \cdot n(a) da]^{1/2} \quad (9)$$

Such a change in the background will produce offsets which simulate noise if the beam moves on the mirror in a random manner at the frequency of the chopper. Hence, as in section II we require relation 4 to hold. This leads to:-

$$[\Delta A / A^2]^{1/2} < [\epsilon_m \cdot \text{NEP} / (2\pi P_m \sqrt{s})] \cdot [e^x - 1] \cdot [\int a^4 Q^2 \cdot n(a) da]^{-1/2} \quad (10a)$$

$$= 1.6 \times 10^{-7} \cdot [\int a^4 Q^2 \cdot n(a) da]^{-1/2} \quad \text{for } T_m = 240\text{K} \quad (10b)$$

$$= 1.3 \times 10^{-5} \cdot [\int a^4 Q^2 \cdot n(a) da]^{-1/2} \quad \text{for } T_m = 77\text{K} \quad (10c)$$

$$= 3.2 \times 10^{-44} \cdot [\int a^4 Q^2 \cdot n(a) da]^{-1/2} \quad \text{for } T_m = 4\text{K} \quad (10d)$$

The trick now is to estimate $[\int a^4 Q^2 \cdot n(a) da]^{-1/2}$.

Dinger and Wooden (see report in Appendix C) examined the dust contamination level on test mirrors placed in the KAO telescope cavity during flights. Their Figure 1 implies that:-

$$\int_d^\infty n(a)da = N_0 d^{-\alpha} \quad \text{for } a > 1\mu\text{m} \quad (11)$$

where $\alpha = 2.5$ and $N_0 = 10^{-5}$ (for "a" in cm). This implies that

$$n(a) = \alpha N_0 a^{-(\alpha+1)} \quad \text{for } a > 1\mu\text{m} \quad (12a)$$

$$= 2.5 \times 10^{-5} a^{-3.5} \quad (\text{for "a" in cm}) \quad (12b)$$

The emissivity of a dust grain, Q , depends on the dust grain's size and composition, and the wavelength of the radiation being observed. In general $Q=1$ for grains that are considerably larger than the wavelength being observed (in this case $\lambda=30\mu\text{m}$) and Q varies as $(a/\lambda)^2$ for grains with radii smaller than the wavelength of observation. We have used this generalization to construct a plausible function, $Q(a)$, for $\lambda=30\mu\text{m}$, as follows:-

$$\begin{aligned} Q(a) &= (a/3 \times 10^{-3})^2 (0.3) & a < 3 \times 10^{-3} \text{ cm} \\ &= (a/10^{-2}) & 3 \times 10^{-3} \leq a \leq 10^{-2} \text{ cm} \\ &= 1 & a \geq 10^{-2} \text{ cm} \end{aligned}$$

With $n(a)$ given by expression 12b, the fraction of the area on a mirror covered by dust is:-

$$\int \pi a^2 n(a) da = 0.015 \quad (13)$$

where we only consider grains with radii greater than $1\mu\text{m}$, since it is only these grains which contribute to the areal density of the dust (see Figure 2 of Dinger and Wooden). And for the values of $Q(a)$ given above, the optical fraction of the area covered by dust is:-

$$[1/\epsilon_m] \int \pi a^2 Q(a) n(a) da = 0.04 \quad (14)$$

[This seems to be a very clean mirror to me.(?!)]

Using the above values for $Q(a)$ and $n(a)$, we can now estimate the integral term in equations 10:-

$$[\int a^4 Q^2(a) n(a) da]^{-1/2} = 1.4 \times 10^3 \quad (15)$$

Thus equation 10 becomes (A in cm^2) :-

$$[\Delta A/A] < (4.8 \times 10^{-8}) A \quad \text{for } T_m = 240\text{K} \quad (16a)$$

$$< (3.4 \times 10^{-4}) A \quad \text{for } T_m = 77\text{K} \quad (16b)$$

$$< (2.0 \times 10^{95}) A !! \quad \text{for } T_m = 4\text{K} \quad (16c)$$

Once again helium cooled mirrors are not a problem.

III. Summary

Let us compare the requirements given by equations (6a) and (16a) for a "2.5 meter class" mirror at 240K where the temperature gradient, $[\Delta T/T_m]$, in equation 6 equals 0.01. Equation 6a then gives

$$|[\Delta A/A_m]| < 4.4 \times 10^{-4}$$

whereas equation 16a gives

$$|[\Delta A/A_m]| < 2.7 \times 10^{-3}.$$

For this case, the requirement caused by the temperature gradient on the mirror is stricter than the requirement caused by the dust on the mirror. However, the requirement caused by dust contamination will become stricter for a dirtier or smaller mirror; for a 1 meter mirror, for example, both the above requirements would be comparable. (The dust contamination level used in section II is close to the level defined by MIL-STD-1246A class 750 - 1000 and, according to Dinger and Wooden, is what we should expect on an airborne observatory.) This discussion shows that both Equation 6 and Equation 16 should be used to determine if background fluctuations are a problem.

An extension of the above example is that Equation 6 can be used to put a requirement on a chopping secondary mirror for a Cassegrain telescope design. The SOFIA baseline design has $[\Delta A_{\text{chop}}/A_m] = 0.01$ for the maximum chopped beam motion on the primary. This beam motion, if perfectly executed, will only introduce a dc-offset which can be removed by a nodding technique (see Davidson and Erickson report), but if there is a random beam motion also associated with the chopping, then noise will be introduced. The above requirements for a 2.5 meter class mirror imply that the random component of this chopped beam motion (ie., the component which simulates noise) must be such that $[\Delta A_{\text{random}}/A_m]$ is less than 4% of $[\Delta A_{\text{chop}}/A_m]$. (Of course, random motion need not be accidental; such a motion could be generated in an active optics system.)

It should be noted here that the above discussion on mirrors would also hold for a window or lens. For such optics, the value for the emissivity, ϵ_m , would be close to 0.2 instead of the value of 0.05 used in the equations of sections II and III. With this different emissivity, equations 6 would become:-

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 1.1 \times 10^{-6} \quad T_m = 240K \quad (17a)$$

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 3.2 \times 10^{-5} \quad T_m = 77K \quad (17b)$$

$$|[\Delta T/T_m] \cdot [\Delta A/A_m]| < 4.2 \times 10^{-4} \quad T_m = 4K \quad (17c)$$

Equations 16, however, would remain unchanged (see Equation 10a). In other words, the effects of temperature gradients are amplified for windows and lenses, but the effects of dust contamination (which depends on the dust, not the surface on which the dust lies) remain unchanged. [As an aside, however, warm lenses are rarely used in an IR optics system since they have high emissivities and so high thermal power emission, which translates to high thermal noise. Hence, the overall NEP of the telescope/sky would increase, which would be very undesirable. Such an unfortunate increase in the NEP would of course relax the above requirements (17) through Equation 4, but the overall system sensitivity would be reduced.]

There is another source of background noise due to beam motion not yet discussed in this report. This source is discussed in the report by Wright, although unfortunately not in a generic way.

Wright discovered for a particular design of the LDR telescope which had the beam passing through an aperture at an image of the sky, that the beam's sidelobes (a diffraction effect) intersected the edge of this aperture when the beam moved by operational amounts, resulting in a fluctuating signal in the sidelobes at the 0.8% P_m level. Worse still, these fluctuations were not confined to the sidelobes, reflections from the edge of the aperture to other mirrors in the telescope system reflected a sizable fraction of these sidelobe fluctuations into the main-beam, causing main-beam power fluctuations about 400 times larger than Equation 4 would allow. This serious problem alone made that particular telescope design for the LDR unacceptable. Thus when looking at telescope designs, diffraction and scattering effects must be considered, especially through apertures (including windows).

Finally, in order to estimate the background noise problems associated with a particular telescope design, the following questions need to be answered before the equations in this report can be used:-

- 1) On which mirrors will the beam move and by how much?
- 2) What are the expected values of $[\Delta A/A_m]$ for each of the mirrors?
- 3) What are the temperatures of these mirrors?
- 4) How well can temperature gradients and dust contamination be controlled on these mirrors?
and
- (5) Are there apertures or telescope structures which could cause background noise with beam motion?

(The last question (based on the Wright result) goes beyond the study in this report, but should probably be the subject of a future SOFIA report.)